22107306

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 6 May 2010 (morning)

1 hour 30 minutes

Candidate session number

| 0 | 0 | | | | | | | | |

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

The following table gives the examination grades for 120 students.

Grade	Number of students	Cumulative frequency
1	9	9
2	25	34
3	35	p
4	q	109
5	11	120

(a) Find the value of

    (i) p;

    (ii) q. *[4 marks]*

(b) Find the mean grade. *[2 marks]*

(c) Write down the standard deviation. *[1 mark]*

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2.   *[Maximum mark: 6]*

An arithmetic sequence, u_1, u_2, u_3..., has $d = 11$ and $u_{27} = 263$.

(a)   Find u_1.   *[2 marks]*

(b)   (i)   Given that $u_n = 516$, find the value of n.

    (ii)   For this value of n, find S_n.   *[4 marks]*

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3. *[Maximum mark: 5]*

Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

(a) Jan tosses the two dice once. Find the probability that she wins a prize. *[3 marks]*

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes. *[2 marks]*

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4. *[Maximum mark: 6]*

Find the term in x^4 in the expansion of $\left(3x^2 - \dfrac{2}{x}\right)^5$.

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5.   *[Maximum mark: 7]*

Consider $f(x) = 2 - x^2$, for $-2 \le x \le 2$ and $g(x) = \sin e^x$, for $-2 \le x \le 2$. The graph of f is given below.
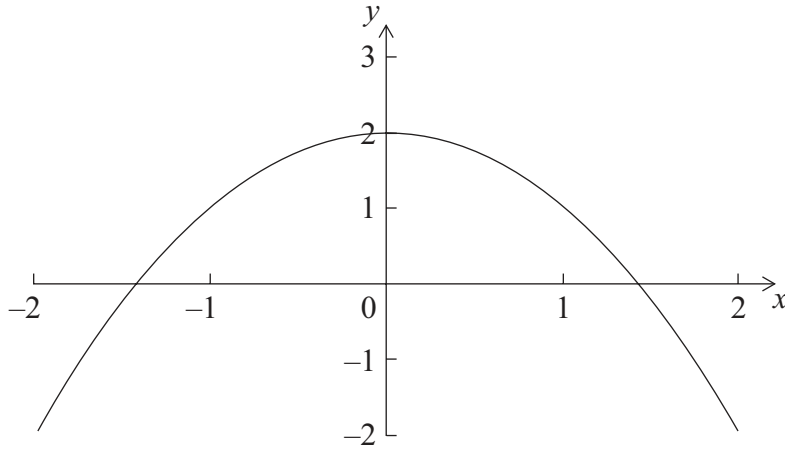
(a)   On the diagram above, sketch the graph of *g*.   *[3 marks]*

(b)   Solve $f(x) = g(x)$.   *[2 marks]*

(c)   Write down the set of values of *x* such that $f(x) > g(x)$.   *[2 marks]*

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6. *[Maximum mark: 6]*

Let $f(x) = e^x \sin 2x + 10$, for $0 \le x \le 4$. Part of the graph of f is given below.

There is an *x*-intercept at the point A, a local maximum point at M, where $x = p$ and a local minimum point at N, where $x = q$.

(a) Write down the *x*-coordinate of A. *[1 mark]*

(b) Find the value of

(i) p;

(ii) q. *[2 marks]*

(c) Find $\int_p^q f(x)\,dx$. Explain why this is not the area of the shaded region. *[3 marks]*

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7. *[Maximum mark: 8]*

The number of bacteria, *n*, in a dish, after *t* minutes is given by $n = 800e^{0.13t}$.

(a) Find the value of *n* when $t = 0$. *[2 marks]*

(b) Find the rate at which *n* is increasing when $t = 15$. *[2 marks]*

(c) After *k* minutes, the rate of increase in *n* is greater than 10 000 bacteria per minute. Find the least value of *k*, where $k \in \mathbb{Z}$. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8.   *[Maximum mark: 15]*

The diagram below shows a circle with centre O and radius 8 cm.
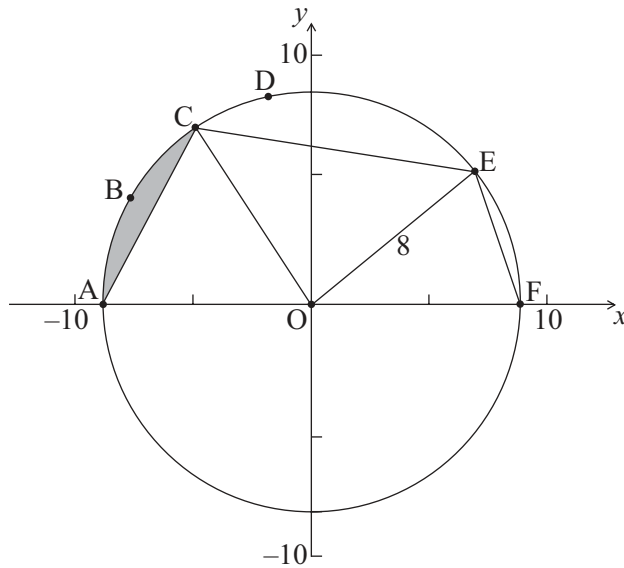
*diagram
not to scale*

The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length
of arc ABC is 6 cm.

(a)   Find the size of angle AOC .   *[2 marks]*

(b)   Hence find the area of the shaded region.   *[6 marks]*

The area of sector OCDE is 45 cm².

(c)   Find the size of angle COE .   *[2 marks]*

(d)   Find EF .   *[5 marks]*

Turn over

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

(a) (i) Write down the coordinates of A.

(ii) Find the speed of the airplane in ms^{-1}. *[4 marks]*

(b) After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds. *[5 marks]*

(c) Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}$, $a \in \mathbb{R}$.

The angle between the flight paths of Airplane 1 and Airplane 2 is $40°$. Find the two values of a. *[7 marks]*

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10. *[Maximum mark: 14]*

Consider $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is given below.
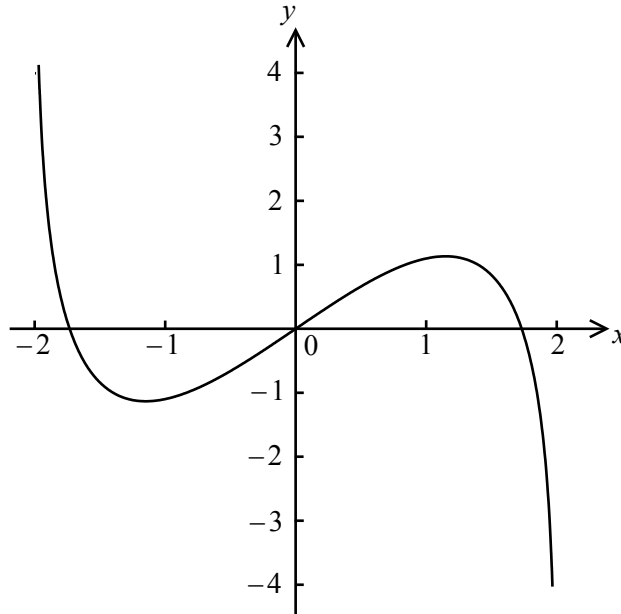
(a) Let P and Q be points on the curve of f where the tangent to the graph of f is parallel to the *x*-axis.

   (i)   Find the *x*-coordinate of P and of Q.

   (ii) Consider $f(x) = k$. Write down all values of k for which there are exactly two solutions. *[5 marks]*

Let $g(x) = x^3 \ln(4 - x^2)$, for $-2 < x < 2$.

(b) Show that $g'(x) = \dfrac{-2x^4}{4 - x^2} + 3x^2 \ln(4 - x^2)$. *[4 marks]*

(c) Sketch the graph of g'. *[2 marks]*

(d) Consider $g'(x) = w$. Write down all values of w for which there are exactly two solutions. *[3 marks]*